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In Defence of the UK Prudential Regulation Authority's Valuation Principles for Equity Release

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Abstract

In recent years the UK Prudential Regulation Authority (PRA) has been involved in a tug of war with equity release firms over the principles to be used in the valuation of equity release mortgages. The PRA proposed a set of such principles in its *Supervisory Statement SS 3/17*. One of these, Principle III ('The present value of deferred possession of a property should be less than the value of immediate possession') has proven to be controversial. We suggest that this controversy is unwarranted and that Principle III can be expected to be valid for properties that are relevant for equity release. We further suggest that the PRA Principles are of major practical importance, because they can be used to establish model-free bounds for any proposed valuations of Equity Release Mortgages and their No-Negative Equity Guarantees. As an illustration of their usefulness, we provide an example which shows that the PRA's own current minimum required deferment rate of 1% can generate valuations that violate the bounds implied by the PRA's Principles.

Key words: Actuarial Science, Black '76 model, CBD mortality models, Equity Release, No Negative Equity Guarantee, Prudential Regulation

JEL Classification: G2, G3

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1. Introduction

In recent years the UK Prudential Regulation Authority (PRA) has been involved in a tug of war with equity release firms over the principles to be used in the valuation of equity release mortgages. The PRA proposed a set of such principles in its *Supervisory Statement SS 3/17*. One of these, Principle III ('The present value of deferred possession of a property should be less than the value of immediate possession') has proven to be controversial. In its *Supervisory Statement SS 3/17* published in July 2017, the UK Prudential Regulation Authority set out certain principles relating to the valuation of Equity Release Mortgage (ERM) portfolios. To quote:

> 1.1 This supervisory statement (SS) sets out the PRA's expectations in respect of firms investing in illiquid, unrated assets within their Solvency II matching adjustment (MA) portfolios. It is relevant to life insurance and reinsurance companies holding or intending to hold unrated assets (including restructured equity release mortgages (ERMs)) in an MA portfolio.[1]

The *Supervisory Statement* continues (para 1.6)

> Chapter 3 … sets out some principles to be applied when assessing the risks from guarantees embedded within ERMs …

The term 'guarantees' refers to the No-Negative Equity Guarantees (NNEGs) that ERM lenders typically grant to borrowers who take out ERM loans. An equity release mortgage (ERM) is a loan made to an older property-owning borrower that is mortgaged against their property and repaid when the borrower permanently leaves it, e.g., on death. A NNEG implies that a borrower never owes the lender more than the value of the property pledged against the loan and constitutes a put option issued by the lender to the borrower. The issue is then how to value these NNEGs. Implicit in *SS 3/17* was the PRA's concern that firms were calibrating their NNEGs using unreasonably low deferment rates.[2] A number of firms at the time were using negative deferment rates and these are strange, because the deferment rate should be equal to the net rental yield and rentals, whether net or gross, are ordinarily and obviously positive. Such deferment rates lead to understated NNEG valuations and overstated ERM valuations, and so boost the apparent i.e. reported profitability of the product.[3]

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[1] The phrase 'restructured equity release mortgages in a Matching Adjustment portfolio' refers to an internally held securitisation in which the firm holds all tranches of the securitisation, and the term 'Matching Adjustment' refers to the regulatory practice of allowing firms to discount liabilities at a rate higher than the risk-free rate.

[2] The deferment rate is the discount rate which when applied to the property price gives the deferment *price*, and the deferment price is the sum to be paid immediately to secure possession of the property at a specified future date.

[3] To elaborate, the approach preferred by academics and the PRA is to use Black '76 (Black, 1976) with the forward house price as the underlying and the deferment rate set equal to the net rental rate. An examples of this approach is Dowd et al. (2019). The approach preferred by equity release actuaries is to use Black '76 but with the underlying given by the projected future house price. An overview of these alternative approaches is provided by Buckner and Dowd (2020).

We have had problems with undervalued guarantees before. A couple of decades ago, there was a scandal surrounding Equitable Life. Founded in 1762, the world's oldest mutual insurer, Equitable Life, pioneered Guaranteed Annuity Rate (GAR) options that offered guaranteed fixed returns. However, it undervalued these options, and in some cases, it didn't value them at all. As a result, the EU wide insurance regulatory system was overhauled to make sure that an Equitable-style fiasco never happened again, the result being the Solvency II insurance regulatory regime. Despite the new regime, there is now a similar issue with the guarantees embedded in equity release mortgages.

We are particularly interested here in Principles II and III, which state:

> [Principle II] The economic value of ERM cash flows cannot be greater than either the value of an equivalent loan without an NNEG or the present value of deferred possession of the property providing collateral;
>
> [Principle III] The present value of deferred possession of property should be less than the value of immediate possession

These two Principles are of major practical importance, because they can be used to establish model-free bounds on any proposed ERM or NNEG valuations. By 'model-free bounds,' we mean bounds that do not depend on any choice of option-pricing model.

To explain, let ERM be the present value of an Equity Release Mortgage loan, L the present value of a risk-free loan of the same notional amount and $NNEG$ be the present value of the NNEG guarantee. ERM can be decomposed as

$$ERM = L - NNEG. \tag{1}$$

The PRA Principles impose an upper bound on ERM. For any given L, the upper bound on ERM implies a lower bound on $NNEG$.

This paper examines these Principles and draws out their implications for ERM and NNEG valuation.

The paper is organised as follows. Section 2 proves Principle II. Section 3 provides a number of alternative demonstrations of the validity of Principle III. Section 4 addresses a number of misconceptions about Principle III. Section 5 discusses the bounds that Principles II and III impose on ERM and NNEG valuations. Section 6 provides an example of how these bounds can be used to identify unreliable valuations and section 7 concludes.

2. Principle II

Principle II states:

> The economic value of ERM cash flows cannot be greater than either the value of an equivalent loan without an NNEG or the present value of deferred possession of the property providing collateral.

i.e.,

(2) $ERM \leq L$ and $ERM \leq PV(F)$

where ERM is the present value of an Equity Release Mortgage loan, L is the present value of a risk-free loan, one which is guaranteed to be repaid in full, and $PV(F)$ is the present value of F, where F is the forward value of the property that provides collateral for the loan.

The principle is proved as follows. First, consider that ERM is by definition the loan value L less the cost of the guarantee *NNEG*, and the latter cannot be negative. It follows that ERM cannot be greater than L. This proves the first proposition of (2) above.

Now consider that the NNEG will only be exercised if the future value of the amount lent exceeds the future value of the property. In this case, the lender receives the future value of the property, in which case the present value of the ERM is equal to the present value of deferred possession. In the alternative case where the NNEG is not exercised, the future value of the amount lent equals the future value of the ERM loan (i.e., *ERM*), which is less than the value of deferred possession. This proves the second proposition of (2) above.

3. Principle III

Principle III states:

> The present value of deferred possession of a property should be less than the value of immediate possession

i.e.,

(10) Deferment house value < spot house value.

The validity of Principle III: demonstration #1

Compare the value of two contracts, one giving immediate possession of the property, the other giving deferred possession when exit occurs. The only difference between these contracts is the value of foregone rights (e.g., to rental income or to use of the property) during the deferment period. The value of these foregone rights should be positive for the residential properties used as collateral for ERMs. It then follows that the present value of deferred possession should be less than the value of immediate possession, i.e., we obtain Principle III.

Principle III thus follows from elementary economics. Why would we not pay less to get less?

The validity of Principle III: demonstration #2

As an alternative demonstration, consider the *deferment house price*, the price that we would agree to pay *today* to take possession of the house in t years' time:

(11) $\quad R_t = S \times e^{-qt}$

where R_t is the deferment house price, S the current or spot house price, and q is the *deferment rate*, defined as the discount rate applied to the current house price to give the *deferment price*.

It can be proved that

(12) $\quad q = d/S = net\ rental\ rate$

where d is the net nominal annual rental. Given that it is reasonable to presume that the net rental $d > 0$, then

(13) $\quad R_t = S \times e^{-qt} < S.$

It is then reasonable to suppose that the deferment house value will be equal to R_t and Principle III follows.[4]

The validity of Principle III: demonstration #3

A longer and more rigorous demonstration goes as follows:

Let w_0, w_1, w_2, ... be the set of net rental yields for a property from now, period 0, to forever. These net rental services are the use-benefits we get from living in a property (e.g., the benefits of having a roof over our heads) or the rental incomes we could obtain by renting the property out.

Let us assume that these are all positive. After all, zero or negative rental yields do not make much sense.

Let A be the set of those net rental yields w, ... for periods 0 to forever.

Let B be the set of net rental yields w_t, w_{t+1} .... from periods t to forever, where $t \geq 1$.

Let C be the set of net rental yields w_0, ... w_{t-1}, for periods 0 to t-1.

[4] As an aside, note that since the interest rate does not enter into (13), then Principle III must hold even if interest rates are negative.

Assume for the moment that the prices of A, B and C all exist.

Since the sets of rentals are positive and hence valuable, then the prices of A, B and C should each be positive. By the law of zero arbitrage, the price of A should also be equal to the sum of the prices of B and C. But since the price of C is positive, it must follow that the price of B < the price of A, i.e., the deferment price must be less than the current price and Principle III is established.

To challenge this conclusion, it is necessary to argue that some of these prices do not exist. Since the price of A is the spot price, then the price of A clearly does exist, so one would have to argue that the prices of B and/or C do not exist.

Let's note to begin with that the empirical basis of any such claim is doubtful. Whilst it is obvious that the prices of B and/or C will rarely exist for some specific property, it is often possible to infer proxy prices for different types of property from comparisons of freehold and leasehold prices and it is these proxy prices that one would use for valuation purposes.[5]

For example, consider a leasehold on a London flat with 99 years to run.[6] The price of this leasehold would typically trade at about 95% of the price of a vacant freehold, and the corresponding freehold, i.e. the right to exclusive possession after 99 years, would trade at about 5% of the vacant value, and gives us the price of possession deferred by 99 years.[7]

But suppose for the sake of argument that some of these prices do not exist and do not have near approximations or proxies in terms of other market prices. In this situation, we simply switch the metric from prices to values and we can establish the validity of Principle III in much the same way as before. For example, if we assume that each net rental yield has a positive value, then it immediately follows that each of A, B and C has positive value, so the value of B must be less than that of A and Principle III follows. Indeed, even if we assume that the current net rental yield is positive and the others are merely non-negative, then Principle III still follows.

To challenge Principle III, one is then left having to argue that net rental yields or the values of net rental yields are negative.

Let's consider possible examples.

One is where the property and the land which it stands are polluted beyond any feasible repair. Chernobyl comes to mind: even if the land could be restored to a usable state, the costs of doing so would be prohibitive. In this case, all q_0, q_1, ... are negative and will remain so. The property and the land itself would then be abandoned. In this case

[5] Typically, one would obtain deferment prices for a particular property by applying rules of thumb to the prices for different property types. These would take account of particular features of a property such as location, parking availability, the size of the garden and so on.

[6] A freehold is an example of a contract for deferred possession and a leasehold is the value of the property less the value of the freehold. Hence the value of the freehold, hence the value of deferred possession, can be inferred from the value of the leasehold.

[7] See the 'relativity graphs': http://www.graphsofrelativity.co.uk/.

Principle III does not hold because the spot and deferred prices are both zero. This type of situation is rare and extreme, however. As another rare example, consider a previously economically attractive property that is now hanging over a cliff after a severe storm. Its spot and deferred prices are now zero and Principle III would not apply. No ERM lender would lend against such properties, however.[8]

A less rare case is where the property is uninhabitable and repair would be uneconomic, but the land itself is valuable. Parts of Detroit come to mind. One might then say that the (current or near current) net rental proceeds were negative, but this situation would not last because the land itself is valuable. The property would be demolished, perhaps after being sold off, and the site redeveloped to restore a positive net rental stream. But such cases are also irrelevant to equity release, because a lender would not make an ERM loan against an uninhabitable property.

A third and more common case is where the property needs repair and repair is economically feasible. The property might not generate any current net rental, but it would be repaired and a positive rental stream restored. This situation is not uncommon, but is still relatively infrequent, in that it does not apply to most properties most of the time.

This third case suggests a property that is uninhabited for a period, because there is no current net rental, so again we would not expect it to be ERM-ed.

The bottom line is that Principle III does not hold for every conceivable property, but we would expect it to hold for a plausibly ERM-able property.

The general case is that most properties most of the time generate a positive net rental stream. Therefore, when looking for a general rule to assess deferment value for equity release purposes, the sensible rule is to assume a positive net rental stream[9] – and a positive net rental stream implies that the deferment value will be less than the current property value.

In short, if the prices of A, B and C all exist and are positive, then the validity of Principle III follows from zero arbitrage. If any of the prices of A, B and/or C do not exist, however, then we can still obtain Principle III by switching over to a rational valuation argument, in which it suffices to argue that the values of A, B and C are all positive because the underlying rentals for ERM-able properties have positive value, because otherwise we would not expect people to be living in them.

The validity of Principle III: demonstration #4, the fiduciary principle

There is also a normative argument that one can call the 'fiduciary principle'. Even where market prices do not exist, accounting principles say that the accountant *should* value economically similar assets in the same way and imply that

[8] This point made, it is entirely possible that some lender previously made an ERM loan against the property, having overlooked its vulnerability to a storm, but that is merely an argument for due diligence.

[9] E.g., we might assume that the average net rental over the expected lifetime of the loan would be positive.

valuation should reflect rational investor preferences. The word 'should' or 'ought' appears, e.g., in IFRS 13 B14a: "Cash flows and discount rates *should* reflect the assumptions that market participants would use when pricing the asset or liability." The fiduciary principle says that an accountant or auditor or some other person, who has an obligation of trust towards a less knowledgeable investor, must value an asset or liability as a *rational knowledgeable investor* (or market participant, or knowledgeable, willing independent person) would. This principle provides a safeguard against interested parties coming back along the lines of "no arbitrage doesn't apply here, so we can make up any price that benefits management, other non-fiduciaries or anyone else we choose." Applying this principle, the accountant, actuary etc. must acknowledge that rental services have positive value and this acknowledgement suffices to establish Principle III.

4. Misconceptions About Principle III

Corporate vs individual valuation of deferred possession

An objection to Principle III is that the Principle may not hold because corporates value deferred possession more than individual houseowners or landlords do. An example of this argument is made by Guy Thomas in a recent posting (Thomas, 2018). In his piece, he acknowledges that the loss of foregone rights (e.g., to income or use of the property) during the deferment period [i.e., the argument underlying Principle III] "appears a reasonable argument" but even so, adds that "there are also reasonable counter-arguments." His key claim is the following:

> … current interests in houses are evidently not attractive to insurers and other institutional investors. Deferred interest might well be more attractive, particularly if in the form of cash-settled financial contracts, so that all the problems of current interests are permanently avoided. Even if a deferred interest is not strictly preferred, *the relative valuation of a deferred interest compared to a current interest seems very likely to be much higher for an insurer than a typical individual owner*. (Our emphasis)
>
> Now if there were a substantial market for deferred interests, the money weight of individuals' preference for current interests versus insurers' preference for deferred interests would determine the relative *market prices* for the two types of interest (i.e. what the PRA calls the 'deferment rate'). But we have the same problem as with the hedging arguments: *the market for deferred interests does not exist on any meaningful scale*. (Our emphasis)

However, there are no legal barriers to a market for deferred interests and such a market already exists in the form of freeholds. If his argument about corporate deferred valuations being above individual deferred valuations were correct, we would also expect insurers to be getting into the deferred property market on the grounds that deferred property ownership is underpriced, and yet he acknowledges that it is not.

In any case, Thomas is comparing one hypothetical non-market valuation (i.e., insurers' valuations of current possession) against another (i.e., their valuations of deferred possession). A comparison of the relative valuations of spot and deferred possession made by a party that is *ex hypothesi* not a major player in the market does not establish (a) anything about the market prices or plausible values for current possession or the market prices or plausible values for deferred possession or any relationship between them. In any case, no such comparison establishes (b) that deferred 'interests' have the negative value necessary to undermine the validity of Principle III.

To make point (a) in a different context, suppose we value a typical stately home as being worth 2 times the value of a typical castle, but the market values a typical stately home as being worth 3 times the value of a typical castle. Our views might be sincerely held, but they are of no relevance if we don't have any portfolios of castles or stately homes and are not in the market trading them. Because we are not in the market trading these things, our views about their relative valuations have no relevance to anyone but ourselves. The only valuations that matter for valuation purposes are those of the market.

IFoA Misconceptions on Principle III

In June 2016, the Institute and Faculty of Actuaries issued "DP 1/16: Equity Release Mortgages: IFoA Response to the Prudential Regulation Authority," its official response to the PRA's earlier Discussion Paper DP 1/16, which had had asked for industry views on ERMs. To quote from this response:

> 33. For the second relationship in paragraph 4.9 [i.e., Principle III] to hold, in theory, there needs to be a deep and liquid market. Otherwise the implication is that the average value of the HPI [House Price Inflation] assumption is less than or equal to the discount rate assumed in the valuation of the NNEG. In practice, the approach to setting the HPI assumption varies significantly from firm to firm.

There are several mistakes here:

- Mistake #1 is that for Principle III "to hold, in theory, there needs to be a deep and liquid market." The validity of Principle III has nothing to do with a deep and liquid market and we have just shown that its validity holds under general conditions.
- Mistake #2 is to suggest that the "average value of the HPI assumption is less than or equal to the discount rate assumed in the valuation of the NNEG." This statement is just plain wrong. The correct statement is that we can assume any HPI we want to, but the assumed value of the HPI is always irrelevant to the valuation of the NNEG.[10]

Para 35 then gives some illustrations of circumstances in which Principle III allegedly might not hold:

[10] This point is developed at length in Buckner and Dowd (2020).

- One is the claim that Principle III "is a statement of 'value' and applies to any individual. However this is not necessarily true in terms of the exchange value." This strange statement is an imaginative addition to the economic theory of value but is unfortunately also wrong. The claim that the Principle III "is a statement of value and applies to any individual" is true, but the corollary is that it also applies to all individuals *including* (and not excluding!) when they engage in trade at market or exchange values.
- Another is the claim that "in a negative yield curve scenario, the relationship (Principle III) would fail as the premise that deferral could lead to a lower present value no longer holds." This statement is a head scratcher but one can see that it must be wrong because the deferment price (or value, makes no difference here) is equal to $S\,e^{-qt}$ and this expression does not include any interest rate or yield, negative or not. To repeat, Principle III depends only on the q rates being positive (or mostly positive) and it is difficult to imagine plausible ERM situations where that would not be the case.

So how could actuaries representing the IFoA make such mistakes? A clue is that the covering letter opens with the following statement:

> The IFoA's *Equity Release Members Interest Group* (ER MIG) and Life Board have been involved in the drafting of this response. The contributors to this response *include members who are actively engaged with use of equity release assets by life insurers*. (My italics)

The IFoA had allowed itself to be used as a mouthpiece for ERM industry practitioners to broadcast their objections aka misunderstandings of Principle III.

But the authors of the IFoA official response to DP 1/16 are not alone in misunderstanding these principles. Consider these passages from a recent Deloitte communiqué on ERMs:

> In our view, the third principle (that future possession of a property cannot be more valuable than current possession) is likely to attract the most future debate.

But Principle III is just elementary economics!

> Very importantly, this principle implies that assumed future house price growth cannot exceed the discount rate applied in the valuation. …

No it does not.

> The PRA expects there to be a positive value associated with possession of a property.

Yes, obviously.

> The practical implication of this is that the assumed house price growth within the NNEG option pricing calculation cannot exceed the discount rate, as this would imply that future possession is more valuable.
>
> This principle therefore effectively sets a cap on firms' house price growth assumptions.

These statements are wrong. Principle III has *no* implications about assumed future house price growth. You can make any assumptions about future house price growth that you like and Principle III would be still be valid.

> We would expect firms investing in ERMs and other direct investments to see an increased level of scrutiny and questioning from the PRA, with the *bar set very high for management's understanding of the valuation of such investments*. (Bulley *et alia*, 2017, our italics)

They are off to a flying start on that one.

We appear to have here another case of 'actuarial judgment' gone awry.

We are reminded of some comments made on this subject by Tim Gordon just over two decades ago (Gordon, 1999). He wrote (p. 4) about the actuarial conviction that "actuarial judgment is the only technique for valuing long-term liabilities" but 'actuarial judgement' produces an answer that "varies enormously depending on which actuary carries out the calculation." He continued:

> actuaries assume that judgmental methods are the only methods available which give sensible answers. What is more, the judgement involved is something which apparently only comes with years of experience. In other words, we claim to know the answer but cannot tell anyone else how to derive it in advance.

The experienced actuary knows it when they see it. Roman augurs had the same skill reading chicken entrails. As he continued further:

> The problem is that the difference that actuarial judgement can make to valuations using the traditional approach is enormous. It means that:
>
> - we are exposed to pressure from clients seeking to move answers in the direction which favours them, and
> - we lose credibility because we are unable to explain precisely how we arrive at an answer.

Actuarial judgment can also lose credibility when it produces answers that are demonstrably wrong.

IFoA ERM Working Party on Principle III

In February 2020, the IFoA took another stab at the Principle III issue in a 'discussion note' issued by its ERM working party (IFoA, 2020). In essence, their view (p. 4) was that 'we would expect the deferment rate of a residential property to usually be positive,' but 'there is no logical necessity for this to always be the case.' This latter phrase is to be understood as suggesting that it is *theoretically possible* that Principle III might not hold for some cases.

We agree.

We gave the example earlier of a property in Chernobyl. In this case, Principle III does not hold because the forward and spot values are identical, i.e., zero. However no equity release lender would lend to properties that are nuclear waste. That is why Chernobyl is not an ERM asset class.

Consequently, any exceptions to Principle III are likely to be irrelevant to equity release, and we should conclude that Principle III makes perfectly good sense for the kinds of properties that equity release lenders might lend against. The WP's emphasis on the *theoretical possibility* of a negative deferment rate is then misdirected, because there is not much point dwelling on theoretical possibilities that are unlikely to *apply to any real world* cases that equity release firms deal with.

The working party has a *deus ex machina* up its sleeve, however: an illiquidity premium (ILP). Unfortunately, this trick doesn't work either.

As the discussion note explains:

> There is a technical argument, presented in recent actuarial ERM valuation research,[11] that the presence of an illiquidity premium in the underlying house price should reduce the cost of the NNEG (note that the illiquidity premium of the residential property is distinct from the illiquidity premium of the mortgage). Specifically, the present value of the house price illiquidity premium that will be earned over the life of the option should be added to the house price that is used in the NNEG valuation. This is equivalent to deducting the house price illiquidity premium from the deferment rate used in the NNEG valuation, implying:
>
> *Deferment rate = Net rental yield – house price illiquidity premium*

Let's play along and calibrate this equation. Our ballpark estimate of the net rental yield is 4.2%.[12] If we accept this estimate for the sake of argument, then we need an illiquidity premium of more than 420 basis points per annum to get to a negative deferment rate. This is an extremely high value for the illiquidity premium and there is no empirical evidence to support it.

[11] 'See Section 5.3.5 and 5.3.6, Jeffery and Smith (2019).'
[12] See Buckner and Dowd (2020, chapter 7).

There is also a deeper problem. 'Technical argument' or no, the working party's equation

(14) $q = net\ rental\ yield - ILP$

is wrong.[13] The correct equation is

(12) $q = net\ rental\ rate = d/S$

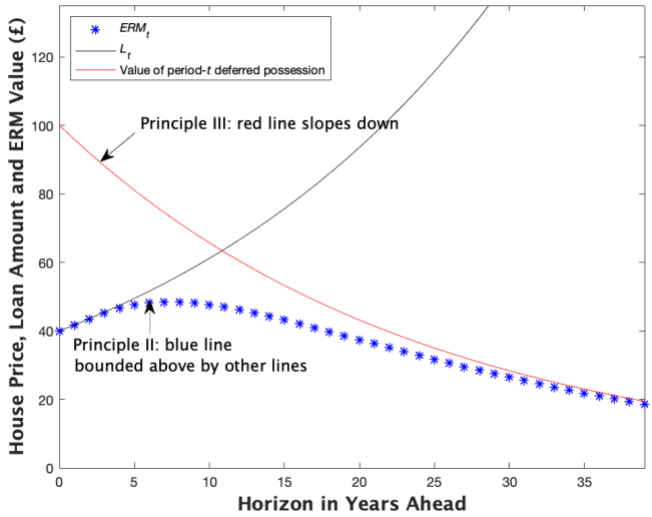
and its correctness is proven in Buckner and Dowd (2020, chapter 7). The underlying reason is that the ILP (or λ), assuming it exists, should appear in the formula for the discount factor $1/(1 + r + \pi + \lambda)$. According to bedrock finance theory in the form, e.g., of the mathematics of the Discount Dividend Model (see Gordon, 1959), λ *can appear nowhere else*. If we accept this discount factor, then Buckner and Dowd (*op. cit.*) shows that the impact of the ILP washes out and the ILP has no effect on the deferment rate. Therefore (12) is correct, and if (12) is correct then unless the ILP is zero, then (15) must be incorrect.

To avoid any misunderstanding, we are *not* saying that the ILP does not exist or does exist but has a zero value. We are not arguing (much) about its size. As far as we care, you can set it as large as you want. We are saying that if the ILP does exist, whatever its size, its impact on the deferment rate gets cancelled out because (12) is always true.

5. Bounds on ERM and NNEG Valuations

To return to the main storyline, the impact of these two Principles is illustrated in Figure 1:

Figure 1: Illustration of Principles II and III



[13] Alternatively, they might have been trying to rewrite one of the fundamental equations of financial mathematics which would be akin to rewriting one of the laws of thermodynamics. That would be brave.

Notes: Based on the baseline assumptions: male aged 70, loan-to-value ratio=40%, risk-free interest rate=1%, loan rate=5.25%, q=4.2% and volatility=20%. Detailed justifications for these calibrations are given in Buckner and Dowd (2020). Exit probabilities are based on the M5-CBD mortality model, Cairns et al. 2009).

Principle II implies that the blue (ERM_t) line must be below both the green (L_t) line and the red (deferred possession) line, and Principle III implies that the red (deferred possession) line should slope downwards.

There is some interesting intuition underlying the Figure:

- For very low horizons, $NNEG_t$ is very out of the money and probability of exercise is very low. Hence the value of the option will be negligible and ERM_t will be indistinguishably close to the value of the loan L_t.
- For long horizons or high t, the option is well into the money and the probability of exercise is high and approaching 1. Therefore, the ERM_t line converges to the deferred house value line for period t.

Underlying these graphs are some elegant mathematics. ERM_t is given by

$$(15) \qquad \begin{aligned} ERM_t &= e^{-rt}L_t - e^{-rt}[L_t N(-d_2) - F_t N(-d_1)] \\ &= e^{-rt}[1 - N(-d_2)]L_t + N(-d_1)e^{-rt}F_t \end{aligned}$$

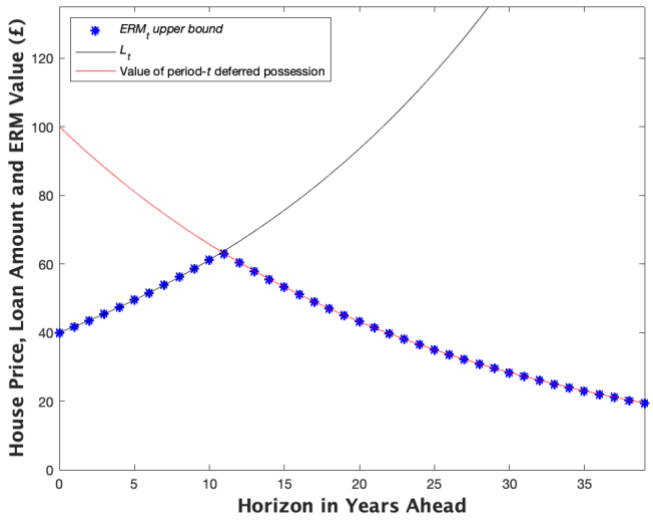
where we have set the deferment price $D_t = e^{-rt}F_t$. From the standard equivalence $N(-x) = 1 - N(x)$, we then get

$$(16) \qquad ERM_t = N(d_2)e^{-rt}L_t + N(-d_1)e^{-rt}F_t.$$

This expression is simpler and reflects the shapes of the curves clearly. As d_2 gets positive, $-d_1$ gets negative, so $N(d_2)$ goes to $1, N(-d_1)$ goes to zero and ERM_t approaches the present value of the loan. As d_2 goes negative, it's the other way round, so the term on the left disappears and the term on the right approaches the deferment price $e^{-rt}F_t$. One sees these bounds at play in Figure 2.

Besides their mathematical elegance, these bounds implied by Principles II and III have a helpful practical use: they are an easily calculated cross-check on any proposed ERM or NNEG valuation. Consider Figure 2, which shows the upper bound for ERM_t made explicit and highlighted in blue.

Figure 2: ERM Upper Bound



Notes: As per Figure 1.

As an aside, if we start with a figure like Figure 1 and let the volatility get small, then it is easy to show that Figure turns into Figure 2 and the message is that the ERM valuation approaches the Principle II upper bound. But if the ERM valuation approaches its upper bound, then the corresponding, e.g., Black 76', option valuation[14] must approach the Principle II NNEG valuation lower bound, i.e., as $\sigma \to 0$, the Black '76 NNEG valuation approaches the Principle II NNEG lower bound.

We can obtain the ERM_t upper bound as the minimum of $e^{-rt}L_t$ and $e^{-rt}F_t$. Note that this upper bound can be estimated using only information about the current house price and LTV (which together give us the current amount loaned), the risk-free rate r, the net rental q, the loan rate l and the exit probabilities. For example, in the baseline case, we estimate the ERM upper bound to be £47.3. So even without estimating ERM or its $NNEG$ *or estimating any underlying option model or calibrating any additional parameters, such as the volatility*, we immediately know that any proposed value of ERM that exceeds £47.3 must be wrong.

But if we can estimate an upper bound for ERM without requiring an option-pricing model or relying on any volatility parameters, then by

$$(1) \qquad ERM = L - NNEG$$

we can also estimate a lower bound for $NNEG$ on the same basis. Given that L = £81.7 in our baseline case, the upper bound ERM estimate of £47.3 implies a $NNEG$ lower bound equal to £34.4. So *even without estimating the NNEG or relying on any NNEG valuation model or any volatility estimate that might go into any such model*, we know that any proposed NNEG value below £34.4 must be wrong.

To cut to the chase, given these various inputs – the assumed age and gender, the assumed house price and LTV, the assumed r, q, and l rates, and the inputted house exit

[14] See Black (1976). For a justification for the Black' 76 model in the ERM context, see also Dowd *et alia* (2019).

probabilities – it is *impossible* to get a NNEG value any lower than £34.4 *whatever option pricing model one might use and regardless of how it might otherwise be calibrated*.

At the risk of repeating ourselves, we would stress that this lower bound NNEG value is *not* dependent on the Black '76 option pricing model. The recent IFoA reply to CP 13/18 released on 28 Sep 2018 made a big deal about how autocorrelation, mean reversion, lack of Geometric Brownian Motion and so forth undermined the validity of the Black '76 option pricing model in this context, and a number of participants at our LSE seminar on 1 October 2018 made similar claims.[15] We would dispute the validity of these claims – not least because they confuse sufficient with necessary conditions for Black-Scholes type valuations to be valid[16] – but even if these claims were all valid, they do not apply to the bounds-based valuation offered here, because that argument is not dependent on any option pricing at all, Black '76 or otherwise.

We also have here a handy cross-check of any proposed NNEG valuation: if any proposed model based on the same input calibrations gives a lower NNEG valuation than either of the NNEG lower bounds, then it must be wrong.

6. Example Application of the PRA's Principles: Is the PRA's Minimum Required q Rate High Enough?

Consider the following practical application of the PRA Principles.

The PRA has sought to counter the NNEG undervaluations and corresponding ERM overvaluations by imposing bounds on the minimum q rates that firms are allowed to use in their valuation models, and the current minimum required q rate is 1%.[17]

The question is then whether this minimum required q rate is high enough to rule out valuations of the PRA bounds?

The answer is no.

Consider the following results based on a minimum required q rate of 1%.

Table 1: Example Valuations vs PRA Bounds

Approach	***NNEG***	***ERM***
Discounted projection (q=1%)	£24.4	£54.4
	***NNEG* lower bound**	***ERM* upper bound**
PRA Principle II bounds	£34.4	£47.3
PRA Principle III bounds	£23.0	£58.8

Notes: As per Figure 1 but with first line based on $q = 1\%$. NNEG option values based on Black '76.

[15] For a write up, see Dowd (2018b).
[16] See Buckner and Dowd (2020, chapter 19; 2020c).
[17] See PRA *Policy Statement 31/18*, December 2018, p. 11.

The valuation for the NNEG is now £24.41 but the Principle II lower bound is £34.4, so $NNEG$ is still below the Principle II lower bound. Correspondingly, the ERM valuation (£54.4) is above the Principle II upper bound (£47.3).

As for the Principle III bounds, it is trivial, since $q = 1\% > q = 0\%$, that the Principle III bounds is satisfied.

In short, for these calibrations, the PRA's minimum requirement of $q = 1\%$ leads to NNEG and ERM valuations that satisfy the Principle III bounds but violate the more demanding Principle II bounds.

7. Conclusions

The Principles set forward by the PRA in Supervisory Statement SS 3/17 mark a major step forward in equity release valuation methodology. Even so, Principle III has been misunderstood by a number of commentators in the equity release sector.[18] However, these objections stem from misunderstandings of basic pricing economics or of option-pricing theory, or from straw man arguments that Principle III does not apply to every conceivable type of property. The response to the latter argument is that we can expect Principle III to hold for properties that are relevant to equity release. That the Principle III-defying counter-examples – properties in Chernobyl or hanging-over-a-cliff, or otherwise uninhabitable or uninhabited – are so contorted that they may as well be on the Moon testifies to why we would expect Principle III to apply to those *mundane* properties that ERM firms are actually willing to lend against. No critic of Principle III has yet been able to give a *single* example of a case where Principle III failed to apply to a property that was relevant to equity release.

Principles II and III are of more than mere academic interest, because they can be used to establish model-free bounds[19] on any proposed ERM and NNEG valuations. Any valuations that violate those bounds should be dismissed. As a practical illustration of their usefulness, we gave an example in which we showed that the PRA's own minimum required deferment rate of 1% can generate valuations that violate these bounds reveal to be impossible. In this case, the PRA's own Principles demonstrate that the PRA's minimum required deferment rate is too low.

We congratulate the PRA on its Principles but encourage it to make more use of them.

[18] We prefer not to speculate on the possibility that some of the pushback against Principle III – such as the IFoA response to DP 1/16 – could be related to the industry's interest in getting the lowest q rates possible, in order to boost ERM valuations and reported short-term profits, and so justify distributions based on those reported profits.

[19] That is, bounds that do not depend on an option-pricing model.

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